制御論第二演習略解

(1) a) $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u, y = \begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$

b) $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, y = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

(2) a), b)ともに，$Y(s) = \frac{1}{(S+3)(S+2)}$

(3) $y(t) = -e^{-3t} x_1(0) + e^{-2t} x_2(0) + \int_0^t (-e^{-3(t-\tau)} + e^{-2(t-\tau)}) u(\tau) d\tau$

(4) a)非可制御・可観測，b)可制御・非可観測

(5) 可制御標準形

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad \boldsymbol{z} = \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix} \boldsymbol{x}$$

可観測標準形

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} -5 & 1 \\ -6 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad \boldsymbol{z} = \begin{pmatrix} -1 & 1 \\ -2 & 3 \end{pmatrix} \boldsymbol{x}$$

(6) $\boldsymbol{f} = \begin{pmatrix} 3 & -1 \end{pmatrix}^T$，$G_{ry}(S) = \frac{1}{(S+1)^2}$

(7) $\boldsymbol{k} = \begin{pmatrix} 1 & -1 \end{pmatrix}^T$

(8) $$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_{m1} \\ \dot{x}_{m2} \end{pmatrix} = \begin{pmatrix} -3 & 0 & 3 & -1 \\ 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_{m1} \\ x_{m2} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} r(t), y(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_{m1} \\ x_{m2} \end{pmatrix}$$

(9)

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_1 - \dot{x}_{m1} \\ \dot{x}_2 - \dot{x}_{m2} \end{pmatrix} = \begin{pmatrix} -3 & 0 & 3 & -1 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_1 - x_{m1} \\ x_2 - x_{m2} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} r(t), y(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_1 - x_{m1} \\ x_2 - x_{m2} \end{pmatrix}$$

(10) $G_{ry}(S)=\dfrac{(S+2)^2}{(S+1)^2(S+2)^2}=\dfrac{1}{(S+1)^2}$

(11)
$$u(t)=\begin{pmatrix}3 & -1\end{pmatrix}\begin{pmatrix}x_{m1}\\ x_{m2}\end{pmatrix}+r(t)$$
$$\begin{pmatrix}\dot{x}_{m1}\\ \dot{x}_{m2}\end{pmatrix}=\begin{pmatrix}0 & -2\\ 1 & -1\end{pmatrix}\begin{pmatrix}x_{m1}\\ x_{m2}\end{pmatrix}+\begin{pmatrix}0\\ 1\end{pmatrix}r(t)+\begin{pmatrix}1\\ -1\end{pmatrix}y(t)$$

(12) $U(S)=\dfrac{-S-6}{S^2+S+2}R(S)+\dfrac{-16S-13}{S^2+S+1}Y(S)$

(13) 制御側$u(t)=(1-\sqrt{2})y(t)$，極$-\sqrt{2}$

(14) $(\sqrt{2}-1)y^2(0)$

(15)
$$u(t)=-3y(t)+4v(t)$$
$$\dot{v}(t)=r(t)-v(t)$$