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1 Introduction

The Traveling Tournament Problem (TTP), established by Easton, Nemhauser and Trick [2], is a sports scheduling problem that abstracts important issues in timetabling. There are some variants on this problem [6], and various studies on TTP have been appeared in recent years. In this paper, we deal with the Constant Distance Traveling Tournament Problem (CDTTP) [5, 7], which is a special class of TTP. We propose a lower bound of the optimal value of CDTTP, and two algorithms that produce feasible solutions whose objective values are close to the proposed lower bound. For some size of instances, one of our algorithms yields optimal solutions.

2 Problem

In this section, we introduce some terminology and definitions needed in this paper, and then describe the constant distance traveling tournament problem (CDTTP).

We are given a set of teams $T = \{1, 2, \ldots, n\}$ (n is even), and each team has its home venue. A game is specified by an ordered pair of teams. A double round-robin tournament is a set of games in which every team plays every other team exactly once at its home venue and once at away (i.e., at the venue of the opponent). Exactly $2(n-1)$ slots (or time periods) are required to play a double round-robin tournament. Each team begins at its home venue and travels to play its games at the chosen venues. Each team then returns (if necessary) to its home venue at the end of a tournament. The number of trips of a team is defined by the number of moves of the team between team venues. Consecutive away games for a team constitute a road trip; consecutive home games are a home stand. The length of a road trip or a home stand is the number of opponents playing against in the road trip/home stand.

The CDTTP is a special class of the original TTP [2] such that the distance between any pair of home venues is equal to one. The CDTTP and its variations are discussed in [5, 7]. The problem CDTTP is defined as follows.

Constant distance traveling tournament problem:

Input: the number of teams, n;
Output: a double round-robin tournament of n teams such that

1. the length of any home stand and that of any road trip is at most three;
2. no repeaters (A at B immediately followed by B at A is prohibited);
3. the total number of trips taken by teams is minimized.

Note that, in the rest of this paper, a double round-robin tournament satisfying the above conditions 1 and 2 is called a *feasible tournament*.

Given a feasible tournament, it is said that a team has a *break* at slot s if it has two consecutive home games or two consecutive away games in slots $s-1$ and s. We also say that a team has a home break (resp., away break) at a game if both of the game and the previous game are at home (resp., away). In a feasible tournament S, the total number of breaks $B(S)$ is defined as the sum of the number of breaks of all the teams. As for the number of trips and the number of breaks, the following lemma has been known.

Lemma 1 (Urrutia and Libeiro [7]). *Let S be a feasible tournament for CDTTP. The total number of trips $D(S)$ and the total number of breaks $B(S)$ have the following relationship:*

$$D(S) = 2n(n-1) - B(S)/2.$$

3 Lower Bound

We prove the following theorem that provides a lower bound of the optimal value of CDTTP.

Theorem 1. *The total number of trips of every feasible tournament of n teams is greater than or equal to* $\mathrm{LB}(n)$ *defined by*

$$\mathrm{LB}(n) \stackrel{\text{def.}}{=} \begin{cases} (4/3)n^2 - n & (n \equiv 0 \mod 3), \\ (4/3)n^2 - (5/6)n - 1 & (n \equiv 1 \mod 3), \\ (4/3)n^2 - (2/3)n & (n \equiv 2 \mod 3). \end{cases}$$

Proof. We first consider the case that $n \equiv 0 \mod 3$. The number of slots satisfies $2(n-1) \equiv 1 \mod 3$. No team can have a break at slot 1 and have three breaks in any consecutive three slots. Thus, the number of breaks for each team is at most $(2/3)(2(n-1)-1) = (4/3)n - 2$. The total number of breaks of every feasible tournament is less than or equal to

$$n((4/3)n - 2) = (4/3)n^2 - 2n.$$

Hence, the total number of trips of every feasible tournament is greater than or equal to

$$2n(n-1) - (1/2)((4/3)n^2 - 2n) = (4/3)n^2 - n.$$

We then consider the case that $n \equiv 1 \mod 3$. The number of slots satisfies $2(n-1) \equiv 0 \mod 3$. No team can have three breaks in consecutive three slots, and hence each team has at most $(2/3)2(n-1) = (4/3)(n-1)$ breaks. Moreover, there are only two types of feasible home-away patterns that have $(4/3)(n-1)$ breaks; that is, HHHAAAHHH $\cdots$ AAA and AAAHHHAAA $\cdots$ HHH. Since no pair of teams has the same home-away pattern [1], at most two teams are possible to have $(4/3)(n-1)$ breaks and other teams have at most $(4/3)(n-1)-1$ breaks. The total number of breaks of every feasible tournament is not more than

$$2(4/3)(n-1) + (n-2)((4/3)(n-1) - 1) = (4/3)n^2 - (7/3)n + 2.$$

The total number of trips of every feasible tournament is greater than or equal to

$$2n(n-1) - (1/2)((4/3)n^2 - (7/3)n + 2) = (4/3)n^2 - (5/6)n - 1.$$

Finally, we consider the case that $n \equiv 2 \mod 3$. The number of home (resp., away) games for each team satisfies $n - 1 \equiv 1 \mod 3$. No team can have a break at the first home (resp., away) game and have three breaks in consecutive three home (resp., away) games. Thus, each team has at most $2(2/3)((n-1)-1) = (4/3)(n-2)$ breaks. The total number of breaks of every feasible tournament is less than or equal to

$$n(4/3)(n-2) = (4/3)n^2 - (8/3)n.$$

The total number of trips of every feasible tournament is greater than or equal to

$$2n(n-1) - (1/2)((4/3)n^2 - (8/3)n) = (4/3)n^2 - (2/3)n. \qquad \square$$

4 Algorithms

In this section, we propose two algorithms, named the Modified Circle Method and the Minimum Break Method, for constructing good feasible tournaments. For each algorithm, we first construct specific single round-robin tournaments, and modify them to double round-robin tournaments.

4.1 Modified Circle Method

We propose the algorithm named *Modified Circle Method* (MCM). Our algorithm has slight differences for each case: (1) $n \equiv 0 \bmod 3$, (2) $n \equiv 1 \bmod 3$, or (3) $n \equiv 2 \bmod 3$. We first explain the whole algorithm for the first case, and mention the differences for other cases.

Denote the set of teams by $T = \{1, 2, \ldots, n\}$. We introduce a directed graph $G^{\mathrm{e}} = (T, A^{\mathrm{e}})$ with a vertex set T and a set of mutually disjoint directed edges

$$A^{\mathrm{e}} \stackrel{\text{def.}}{=} \{(j, n+1-j) \ : \ \lceil j/3 \rceil \text{ is even}, \ 1 \leq j \leq n/2\} \\ \cup \{(n+1-j, j) \ : \ \lceil j/3 \rceil \text{ is odd}, \ 1 \leq j \leq n/2\}.$$

Let $G^{\mathrm{o}} = (T, A^{\mathrm{o}})$ be a directed graph obtained from G^{e} by reversing the direction of the edge between 1 and n. For each $s \in \{1, 2, \ldots, n-1\}$, we define a permutation π^s by $(\pi^s(1), \pi^s(2), \ldots, \pi^s(n)) = (s, s+1, \ldots, n-1, 1, 2, \ldots, s-1, n)$. For any permutation π on T, $G^{\mathrm{e}}(\pi)$ (resp., $G^{\mathrm{o}}(\pi)$) denotes the set of $n/2$ matches satisfying that every directed edge $(u, v) \in A^{\mathrm{e}}$ (resp., A^{o}) corresponds to a match between $\pi(u)$ and $\pi(v)$ held at the home venue of $\pi(v)$. Let X be a single round-robin tournament satisfying that matches in slot s are defined by $G^{\mathrm{o}}(\pi^s)$ (if $s \in \{1, 2, 3\} \bmod 6$) and $G^{\mathrm{e}}(\pi^s)$ (if $s \in \{4, 5, 0\} \bmod 6$).

Consider the case that $n \equiv 0 \bmod 3$. For each $i \in \{1, 2, \ldots, n/3 - 1\}$, we denote a partial schedule of X consisting of a sequence of three slots $(3i-2, 3i-1, 3i)$ by X_i. Moreover, we denote a partial schedule of X consisting of two slots $(n-2, n-1)$ by $X_{n/3}$. Now we construct a double round-robin tournament Y by concatenating above partial schedules as follows: $Y = (X_1, \overline{X_1}, \overline{X_2}, X_2, X_3, \overline{X_3}, \overline{X_4}, X_4, X_5, \ldots, \overline{X_{\frac{n}{3}}}, X_{\frac{n}{3}})$, where $\overline{X_i}$ is a partial schedule obtained from X_i by reversing all venues. Note that the time complexity to construct the tournament Y is $\mathbf{O}(n^2)$ (i.e., MCM runs in linear time of the input and output).

Consider the case that $n \equiv 1 \bmod 3$. We construct a single round-robin tournament X completely same as the above case. For each $i \in \{1, 2, \ldots, (n-1)/3\}$, we denote a partial schedule of X consisting of a sequence of three slots $(3i-2, 3i-1, 3i)$ by X_i. We construct a double round-robin tournament Y by concatenating partial schedules: $Y = (X_1, \overline{X_1}, \overline{X_2}, X_2, X_3, \ldots, X_{\frac{n-1}{3}}, \overline{X_{\frac{n-1}{3}}})$.

Consider the case that $n \equiv 2 \bmod 3$. Let $\tilde{G}^{\mathrm{e}}$ (resp., $\tilde{G}^{\mathrm{o}}$) be a directed graph obtained from G^{e} (resp., G^{o}) by reversing the direction of the edge between $n/2 - 1$ and $n/2 + 2$. We construct a single round-robin tournament X as well as the above cases using directed graphs $\tilde{G}^{\mathrm{e}}$ and $\tilde{G}^{\mathrm{o}}$. For each $i \in \{2, 3, \ldots, (n-2)/3\}$, we denote a partial schedule of X consisting of a sequence of three slots $(3i-3, 3i-2, 3i-1)$ by X_i. We denote a partial schedule of X consisting of two slots $(1, 2)$ by X_1 and two slots $(n-2, n-1)$ by $X_{(n+1)/3}$. We construct a double round-robin tournament Y by concatenating above partial schedules: $Y = (X_1, \overline{X_1}, \overline{X_2}, X_2, X_3, \ldots, X_{\frac{n+1}{3}}, \overline{X_{\frac{n+1}{3}}})$.

Lemma 2. *Y is a feasible double round-robin tournament.*

Proof. It is clear that Y is a double round-robin tournament. Repeater does not appear in the tournament Y; when the game A at B appears in slot s, the game B at A appears in slot $s-3$ or $s+3$ (if a partial schedule of three slots includes slot s), or appears in slot $s-2$ or $s+2$ (if a partial schedule of two slots includes slot s).

We estimate the maximum length of home stand/road trip. We first consider partial schedules of six or four slots $(X_i, \overline{X_i})$ and $(\overline{X_i}, X_i)$. Four consecutive games in these partial schedules must have both of home and away games since they include two games of the same pair of teams. Thus, the maximum length of home stand/road trip in these partial schedules is at most three. We then consider partial schedules of six or five slots (X_i, X_{i+1}) and $(\overline{X_i}, \overline{X_{i+1}})$. Each team has at most three consecutive home/away games in single round-robin tournaments X and $\overline{X}$ (where $\overline{X}$ is a single round-robin tournament obtained from X by reversing all venues). Thus, the maximum length of home stand/road trip in these partial schedules is also at most three. Hence, the length of any home stand and that of any road trip in Y is at most three. □

Lemma 3. *The total number of breaks of double round-robin tournament Y is*

$$B(Y) = \begin{cases} (4/3)n^2 - (8/3)n + 2 & (n \equiv 0 \mod 3), \\ (4/3)n^2 - 3n + (8/3) & (n \equiv 1 \mod 3), \\ (4/3)n^2 - (13/3)n + (10/3) & (n \equiv 2 \mod 3). \end{cases}$$

Proof. We first consider the second slot of a partial schedule X_i that has three slots. If a team has a break (resp., no break) at the second slot of X_i in X, this team has two breaks (resp., no break) at the second slots of X_i and $\overline{X_i}$ in Y. These properties are similar to the third slot of X_i that has three slots and the second slot of X_i that has two slots.

We then consider the first slot of a partial schedule X_i. In order to know the number of breaks at the first slots of X_i and $\overline{X_i}$ in Y, we check the followings: (1) a team has a break or not at the first slot of X_i in X, and (2) a team has different type games (i.e., home and away) at the first and last slots of X_i or not. The number of positive answers for above questions is equal to the number of breaks at the first slots of X_i and $\overline{X_i}$ in Y. We note that the following property makes the checking process easier: the single round-robin tournament X does not have patterns HHHH, AAAA, HAH nor AHA.

Now, we compute the number of breaks in Y. We first consider the case that $n \equiv 0 \bmod 3$. Teams $1, u$ ($u \equiv 0 \bmod 3$) and $n-1$ have $(4/3)n-2$ breaks each; other teams have $(4/3)n-3$ breaks each. In total, we have $(4/3)n^2-(8/3)n+2$ breaks in the double round-robin tournament Y.

We then consider the case that $n \equiv 1 \bmod 3$. Team n has $(4/3)n-(4/3)$ breaks. Teams u ($u \equiv 0 \bmod 3, u < n/2$), $n/2$ and v ($v \equiv 1 \bmod 3, n/2 < v < n$) have $(4/3)n-(7/3)$ breaks each. Other teams have $(4/3)n-(10/3)$ breaks each. Thus, the total number of breaks of Y is $(4/3)n^2-3n+(8/3)$.

We finally consider the case that $n \equiv 2 \bmod 3$. Team n has $(4/3)n-(8/3)$ breaks. Teams 1, u ($u \equiv 0 \bmod 3, u < n/2-2$), $n/2-2, n/2, n/2+2$ and v ($v \equiv 2 \bmod 3, n/2+2 < v < n$) have $(4/3)n-(11/3)$ breaks each. Other teams have $(4/3)n-(14/3)$ breaks each. Hence, the total number of breaks of Y

is $(4/3)n^2 - (13/3)n + (10/3)$. □

Let the total number of trips of a feasible tournament Y be $D(Y)$. Using Theorem 1 and Lemmas 1, 2 and 3, we have the following theorem on the Modified Circle Method.

Theorem 2. *The Modified Circle Method produces a feasible tournament Y such that*

$$D(Y) = \begin{cases} (4/3)n^2 - (2/3)n - 1 \quad = \mathrm{LB}(n) + (1/3)n - 1 & (n \equiv 0 \mod 3), \\ (4/3)n^2 - (1/2)n - 4/3 = \mathrm{LB}(n) + (1/3)n - 1/3 & (n \equiv 1 \mod 3), \\ (4/3)n^2 + (1/6)n - 5/3 = \mathrm{LB}(n) + (5/6)n - 5/3 & (n \equiv 2 \mod 3). \end{cases}$$

4.2 Minimum Break Method

We propose the algorithm named *Minimum Break Method* (MBM). Before starting the explanation of MBM, see the following lemmas as for the number of breaks in a single round-robin tournament.

Lemma 4 (de Werra [1]). *For any single round-robin tournament of n teams, the number of breaks is not less than $n - 2$. There exists a single round-robin tournament that has $n - 2$ breaks for any even n.*

Lemma 5 (Miyashiro and Matsui [4]). *For any single round-robin tournament of n teams, the number of breaks is not more than $n^2 - 3n + 2$. There exists a single round-robin tournament that has $n^2 - 3n + 2$ breaks for any even n.*

Let X be a single round-robin tournament satisfying the following conditions:

(C1) the number of total breaks $B(X)$ is equal to $n - 2$;
(C2) no team has a break at each slot $s \in \{1, 4\} \bmod 6$ (if $n \in \{0, 1\} \bmod 3$) or $s \in \{0, 3\} \bmod 6$ (if $n \equiv 2 \bmod 3$).

It is known that any single round-robin tournament must have at least $n - 2$ breaks; hence, X is a tournament with the minimum number of breaks. Here, we have two open problems: (1) such single round-robin tournament X exists or not, and (2) if X exists, an efficient algorithm to construct X exists or not. We predict that a single round-robin tournament X that satisfies Conditions (C1) and (C2) exists for any even n.

In order to obtain a single round-robin tournament satisfying Conditions (C1) and (C2), we adopt the following strategy. We first replace Condition (C2) to another condition:

(C1) the number of total breaks $B(X)$ is equal to $n - 2$;
(C2') no team has a break at each slot $s \in \{1, 2, 4\} \bmod 6$ (if $n \in \{0, 1\} \bmod 3$) or $s \in \{0, 1, 3\} \bmod 6$ (if $n \equiv 2 \bmod 3$), and exactly two teams have a break at each slot $s \in \{0, 3, 5\} \bmod 6$ (if $n \in \{0, 1\} \bmod 3$) or $s \in \{2, 4, 5\} \bmod 6$ (if $n \equiv 2 \bmod 3$).

If a tournament satisfies Conditions (C1) and (C2'), it always satisfies Conditions

(C1) and (C2). We then try to find a single round-robin tournament satisfying Conditions (C1) and (C2'). When $n \leq 50$, we have obtained such single round-robin tournaments by solving integer programming problems (e.g., see [3]).

Now we construct another single round-robin tournament X' from X by reversing venues for each even slot. This tournament X' satisfies that exactly two teams have $n-2$ breaks and other teams have $n-3$ breaks; hence, X' is a single round-robin tournament with the maximum number of breaks. Moreover, X' satisfies that every team has a break at slot s satisfying $s \equiv 1 \bmod 3$ (if $n \in \{0,1\} \bmod 3$, except for $s = 1$) or $s \equiv 0 \bmod 3$ (if $n \equiv 2 \bmod 3$).

We then construct a double round-robin tournament Y' with X'. If $n \equiv 0 \bmod 3$, we denote a partial schedule of X' consisting of a sequence of three slots $(3i-2, 3i-1, 3i)$ by X'_i for each $i \in \{1, 2, \dots, (n/3)-1\}$, and we denote a partial schedule of X' consisting of two slots $(n-2, n-1)$ by $X'_{n/3}$. If $n \equiv 1 \bmod 3$, for each $i \in \{1, 2, \dots, (n-1)/3\}$, we denote a partial schedule of X' consisting of a sequence of three slots $(3i-2, 3i-1, 3i)$ by X'_i. If $n \equiv 2 \bmod 3$, we denote a partial schedule of X' consisting of a sequence of three slots $(3i-3, 3i-2, 3i-1)$ by X'_i for each $i \in \{2, 3, \dots, (n-2)/3\}$, and we denote a partial schedule of X' consisting of two slots $(1, 2)$ by X'_1 and two slots $(n-2, n-1)$ by $X'_{(n+1)/3}$. Now we construct a double round-robin tournament Y' by concatenating above partial schedules as follows: $Y' = (X'_1, \overline{X'_1}, X'_2, \overline{X'_2}, X'_3, \overline{X'_3}, X'_4, \overline{X'_4}, X'_5, \dots)$, where $\overline{X'_i}$ is a partial schedule obtained from X'_i by reversing all venues.

Lemma 6. *Y' is a feasible double round-robin tournament.*

Proof. It is clear that Y' is a double round-robin tournament. Repeater does not appear in the tournament Y'; when the game A at B appears in slot s, the game B at A appears in slot $s-3$ or $s+3$ (if a partial schedule of three slots includes slot s), or appears in slot $s-2$ or $s+2$ (if a partial schedule of two slots includes slot s). We then show that the length of any home stand and that of any road trip is at most three. We first consider a partial schedule of six or four slots $(X'_i, \overline{X'_i})$: four consecutive games in this partial schedule must have both of home and away games since they include two games of the same pair of teams. No team has a break at the first slot of X'_i in the tournament Y', thus four or more consecutive home/away games do not appear in a partial schedule of six or five slots $(\overline{X'_i}, X'_{i+1})$. Hence, the length of any home stand and that of any road trip is at most three. □

Lemma 7. *The total number of breaks of double round-robin tournament Y' is*

$$B(Y') = \begin{cases} (4/3)n^2 - 3n + 2 & (n \equiv 0 \mod 3), \\ (4/3)n^2 - (7/3)n + 2 & (n \equiv 1 \mod 3), \\ (4/3)n^2 - (11/3)n + 2 & (n \equiv 2 \mod 3). \end{cases}$$

Proof. The single round-robin tournament X' satisfies that exactly two teams have $n-2$ breaks and other teams have $n-3$ breaks. Moreover, X' satisfies that every team has a break at slot s satisfying $s \equiv 1 \bmod 3$ (if $n \in \{0,1\} \bmod 3$, except for $s = 1$) or $s \equiv 0 \bmod 3$ (if $n \equiv 2 \bmod 3$). Thus, we have the following properties on X'_i: (1) every team has a break at the first slot of each X'_i in X' except for X'_1, (2) no team has a break at the first slot of each X'_i in Y', and (3) every team has one or two breaks at the second and third slots (resp., zero or one break at the second slot) of each X'_i of three slots (resp., two slots).

Consider a partial schedule X'_i of three slots. If a team has two breaks at the second and third slots in X'_i (i.e., HHH or AAA), this team has four breaks in a partial schedule $(X'_i, \overline{X'_i})$ in Y' (i.e., HHHAAA or AAAHHH). If a team has just one break at the second and third slots in X'_i (i.e., HHA, HAA, AAH or AHH), this team has three breaks in a partial schedule $(X'_i, \overline{X'_i})$ in Y' (i.e., HHAAAH, HAAAHH, AAHHHA or AHHHAA). Consider a partial schedule X'_i of two slots. If a team has one break at the second slot in X'_i (i.e., HH or AA), this team has two breaks in a partial schedule $(X'_i, \overline{X'_i})$ in Y' (i.e., HHAA or AAHH). If a team does not have a break at the second slot in X'_i (i.e., HA or AH), this team has one break in a partial schedule $(X'_i, \overline{X'_i})$ in Y' (i.e., HAAH or AHHA).

Now, we can compute the number of breaks for each team in a double round-robin tournament Y' as follows:

$$\text{(number of partial schedules of three slots)} + \text{(number of breaks)} + 1,$$

and we can compute the total number of breaks of double round-robin tournament Y'. □

Using Theorem 1 and Lemmas 1, 6 and 7, we have the following theorem for the Minimum Break Method.

Theorem 3. *If there is a single round-robin tournament satisfying Conditions (C1) and (C2), the Minimum Break Method produces a feasible tournament Y' such that*

$$D(Y') = \begin{cases} (4/3)n^2 - (1/2)n - 1 = \mathrm{LB}(n) + (1/2)n - 1 & (n \equiv 0 \mod 3), \\ (4/3)n^2 - (5/6)n - 1 = \mathrm{LB}(n) & (n \equiv 1 \mod 3), \\ (4/3)n^2 - (1/6)n - 1 = \mathrm{LB}(n) + (1/2)n - 1 & (n \equiv 2 \mod 3). \end{cases}$$

Note that, as mentioned before, we obtained single round-robin tournaments satisfying Conditions (C1) and (C2) for $n \le 50$. Thus, using MBM with those single round-robin tournaments, we obtained feasible double round-robin tournaments Y' for instances up to 50 teams.

5 Results

We summarize our results on CDTTP appeared in this paper. For instances with $n \equiv 0$ mod 3 teams, MCM gives better solutions compared to MBM. In contrast, for instances with $n \in \{1, 2\}$ mod 3 teams, MBM performs better though it needs single round-robin tournaments satisfying Conditions (C1) and (C2). In addition, when $n \equiv 1$ mod 3, MBM yields a solution that attains LB(n), i.e., an optimal solution. We obtained single round-robin tournaments satisfying Conditions (C1)

Table 1. Results for $16 \le n \le 24$.

n	LB(n)	MCM	MBM	known
16	327	332	*327	327
18	414	419	426	417
20	520	535	529	520
22	626	633	*626	628
24	744	751	755	750

*: our solutions that attain the lower bound LB(n),
known: the known best solutions in [6], as of August 2006.

and (C2) for instances with up to 50 teams. Table 1 shows the results for $16 \le n \le 24$: for $n = 16, 22$, MBM gave optimal solutions; for $n = 20$, our lower bound showed that the known best solution is an optimal solution.

6 Conclusions

In this paper, we have considered the constant distance traveling tournament problem (CDTTP), a simple variant on TTP. We computed a lower bound of the optimal value of CDTTP. The proposed lower bound seems effective; we could show that some existing and our double round-robin tournaments are optimal using the proposed lower bound. We also proposed two algorithms to construct feasible tournaments. Our algorithms first construct single round-robin tournaments, divide them to partial schedules with two or three slots, and concatenate them to make double round-robin tournaments. The Modified Circle Method is a simple heuristic algorithm that runs in linear time of the size of the timetable. The Minimum Break Method produces optimal solutions for instance with $n \equiv 1 \mod 3$ teams, if we can obtain single round-robin tournaments satisfying some constraints.

Our future work is to improve algorithms to construct feasible tournaments for the CDTTP. If we could have positive conclusions for the open problems appeared in Section 4.2, MBM becomes a polynomial time algorithm. Another work for future is to tackle other variants on TTP (e.g., the circular distance traveling tournament problem).

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