Success Guaranteed Routing in Almost Delaunay Planar Nets for Wireless Sensor Communication

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Abstract. This paper proposes a routing strategy for wireless sensor networks that is valid even when the sensor nodes are distributed nonuniformly. Energy efficiency is the most important consideration in wireless sensor networks; hence, global communication should be achieved by combinations of local messages. We first propose a new underlying graph called the almost Delaunay planar net, which can be calculated from information local to the sensor nodes. We statistically analyze the efficiency of existing routing strategies in the almost Delaunay planar net. We also propose a new routing strategy that always guarantees the reachability on planar graphs, including the almost Delaunay planar net. We show that the proposed routing strategy exhibits good performance if the underlying graph in the sensor network is the almost Delaunay planar net.

Keywords: Wireless sensor network, routing strategy, Delaunay triangulation, Voronoi polygon, cluster head.

1 Introduction

A wireless sensor network is a network composed of sensor nodes connected by wireless communication links. A typical application of a wireless sensor network is environmental monitoring, where randomly distributed sensors communicate with each other to form an autonomous network relaying sensor data to a monitoring center. One of the primary technical concerns here is energy efficiency, because the preinstalled battery power in the sensors is limited. Therefore, we want to use only short-distance communications instead of direct communication with the monitoring center. To achieve this, we first determine an underlying graph structure by selecting relatively short links between sensor nodes, and then use this graph to relay messages from sensors to the monitoring center by wireless communication.

Various underlying graphs have been proposed and studied. The most basic concepts in this context are Delaunay triangulation and the unit disk graph [6, 9, 14]. Delaunay triangulation is a good approximation of Euclidean space in the sense that the shortest path in this graph is within a constant ratio of the Euclidean distance between any two nodes, and it can be constructed efficiently. Bose et al. [1] proposed a local routing algorithm that can find a route to the monitoring center whose length is within a bounded ratio of the shortest-path length of the Delaunay triangulation. Some subgraphs of the Delaunay triangulation, including the Gabriel graph, and the relative neighborhood graph, are also studied as underlying graphs for sensor networks [11]. Although the Delaunay triangulation and related graphs have many good properties, we need global computation to construct them. Global computation is energetically expensive for sensor networks, and so we want to avoid it because available energy is limited.

On the other hand, the unit disk graph is generated by connecting all pairs of nodes within a unit distance (corresponding to the maximum distance for wireless communication). This graph can be constructed locally, but usually it contains too many edges for sensor networks. Hence, subgraphs of the unit disk graph are usually used. One simple idea might be to use those edges that are contained in both the Delaunay triangulation and the unit disk graph. However, this is also difficult because recognition of Delaunay edges requires global information. Therefore, locally computable variants have been proposed. They include localized Delaunay triangulations [9], partial Delaunay triangulations [10], almost Delaunay triangulations [3, 4, 5], restricted Delaunay graphs [2], constrained Delaunay triangulations [11], and planarized local Delaunay triangulations [14].

In a restricted Delaunay graph [2] construction, a hierarchical clustering algorithm is used to select a small subset of the sensor nodes, called cluster heads. Each node in the sensor network can communicate directly to some cluster heads. A restricted Delaunay graph of a set of points in the plane is a planar graph and contains all the Delaunay edges with length ≤ 1. From the restricted Delaunay graph, a subset of nodes is selected as the
cluster heads so that the remaining nodes are visible to at least one cluster head. A hierarchical algorithm is used for several rounds to select the cluster heads, which is only a constant factor more than the minimum possible number of cluster heads. Each noncluster head node (called a client) is assigned to a unique cluster head close to it. Pairs of cluster heads that may communicate with each other via their clients are identified. For each such pair, one pair of clients is selected, called gateways, to enable communication between the cluster head pair. This reduces the routing problem over the whole graph to routing between cluster heads and gateways.

Li et al. [9] proposed localized Delaunay triangulation, which generates Delaunay triangulation with local information only. The locality is defined by $k$-step neighbor information for a positive constant $k$. Localized Delaunay triangulation is a planar graph for $k \geq 2$. A triangle is called a localized Delaunay triangle if it is a $k$-localized Delaunay triangle for some constant integer $k \geq 1$. In localized Delaunay triangulation generation, sensor nodes should exchange their information with up to $k$-step neighbors. Each node has the information of all its $k$-step neighbors. Once we fix the underlying graph, we have to construct a strategy to relay messages from the sensor nodes to the monitoring center using only local computation. Many greedy strategies have been proposed. They include compass routing [7], random compass routing [11], most forward routing [11], nearest neighbor routing [11], farthest neighbor routing [11], greedy perimeter stateless routing [8], and face routing [1].

To achieve good performance of the sensor network, we need to select an appropriate pair of the underlying graph and routing strategy. However, it is not easy to analyze the performance theoretically, and so various experimental studies have been done by many authors [1, 2, 3, 9, 11, 12, 13, 14, 15].

Most of the previous studies are based on the assumption that sensors are distributed uniformly. However, this assumption is often unrealistic. For example, suppose that we want to collect weather data in a mountain area containing a large lake and the sensors in the lake do not work. Then, the resulting sensor network behaves differently from the uniform case.

In this paper, we study underlying graphs and routing strategies for the sensor networks where the distribution of the sensor nodes is not always uniform. Our main concern is how to guarantee the reachability of messages from any nodes to the monitoring center without increasing energy consumption. From this point of view, we propose an underlying graph that is similar to Delaunay triangulation in some sense, which we call an almost Delaunay planar net, and a routing strategy that is a mixture of three existing methods. We give a theoretical proof of the reachability of this strategy and observe the efficiency of the strategy by computational experiments.

## 2 Almost Delaunay Planar Net

Suppose that we are given $n$ sensor nodes in a two-dimensional sensing area. The length of an edge $(u,v)$ is defined as the Euclidean distance of its two end nodes $u$ and $v$, and denoted by $||uv||$. The objective of this section is to design an underlying graph that includes only short edges and can be constructed with local information for each sensor node. Let $r$ be the transmission range for all nodes and each node $u$ can communicate with nodes $v$ if and only if $||uv||$ is at most $r$. Let $E_{\text{MSTMAX}}$ denote the length of the longest edge in the minimum spanning tree of sensor nodes, the transmission range $r$ should be $E_{\text{MSTMAX}}$ or larger for connectivity. We note that estimation of the accurate value of $E_{\text{MSTMAX}}$ needs global computation. Given the number of nodes $n$ and the sensing area $A$, we approximate the value of $E_{\text{MSTMAX}}$ by computer simulation.

We now construct an underlying graph called an almost Delaunay planar net, defined constructively by Algorithm 1 below. It contains all the edges of the Delaunay triangulation whose lengths are less than or equal to a threshold value $r$. It does not include any edge that is longer than the threshold value. It also does not have any edge crosses, i.e., the resulting graph is planar. This graph is similar to the restricted Delaunay graph [2], the localized Delaunay graph [9], and the almost Delaunay triangulation [3, 4]. In particular, Li et al. [9] defined the $k$-localized Delaunay triangulation graph, which is generated with only $k$-step neighbor information from a given unit disc graph. Coincidently, our almost Delaunay planar net is a 2-localized Delaunay graph. Li et al. [9] did not specifically give any routing strategy for the $k$-localized Delaunay graph. They only compared the individual standard routing strategies such as compass routing, random compass routing, greedy routing, and most forward routing in their graph. In Section 3 we give a routing strategy that is not only suitable for our proposed almost Delaunay planar net but also suitable for any other planar graph. Finally, we give some comparisons with different routing strategies in the almost Delaunay planar net. Formally, the almost Delaunay planar net is defined constructively by the following algorithm.

**Algorithm 1 (almost Delaunay planar net)**

**Input:** sensor nodes distributed in the plane, and a positive constant $r$ satisfying $r \geq E_{\text{MSTMAX}}$.

**Output:** almost Delaunay planar net.
Procedure:

**Step 1:** (Generating a neighborhood list for each sensor node.)

(a) Send a `hello` message within the transmission range $r$ and wait for the reply.
(b) For every node, upon receiving a `hello` message, respond to the sender with its location information.
(c) Construct a neighborhood list according to the received acknowledgments.

**Step 2:** (Constructing the Delaunay triangulation with their neighbors.)

For each node $u$, calculate the Voronoi polygon formed by the perpendicular bisectors of each line segment $(u, v_i)$, where $v_i$ is a neighbor of $u$. Connect $u$ with the neighbor nodes that have common edges in the Voronoi polygon. Let this graph be $G_1$.

**Step 3:** (Eliminating unnecessary edges.)

Each node sends its neighbor information to all its neighbors. If two end nodes have a differing opinion for their common edge, then it is not a Delaunay edge and will be removed. If there is an edge-crossing in $G_1$, then at least one of the four nodes of the crossing edges has information of all the other three nodes (see Property 1). This node can check which edge violates the local Delaunay property and remove the illegal edge.

**Property 1.** If two edges cross each other after Step 2 in Algorithm 1, then one of the four nodes is a neighbor of all the other three nodes.

**Proof:** Let us consider two edges $(a, b)$ and $(c, d)$, which intersect each other at $x$, as in Fig. 1.

![Fig. 1 Edge crossing.](image)

Then,

$$
\|ax\| + \|xc\| \geq \|ac\|,
\|bx\| + \|xd\| \geq \|bd\|.
$$

So,

$$
\|ax\| + \|xc\| + \|bx\| + \|xd\| \geq \|ac\| + \|bd\|,
\|ab\| + \|cd\| \geq \|ac\| + \|bd\|.
$$

In our algorithm, all the edges have a length of at most $r$, i.e., both $\|ab\|$ and $\|cd\|$ are at most $r$. This means that at least one of $\|ac\|$ or $\|bd\|$ is less than or equal to $r$. Similarly, we can show that either $\|ad\|$ or $\|bc\|$ is at most $r$. In all possible cases, there is at least one node from which all the remaining nodes are located within distance $r$. 

Let us check properties of the almost Delaunay planar net. Clearly, this graph can be constructed locally. More precisely, each node uses only information from within a 2-hop neighborhood. In Step 1, the algorithm constructs the unit disk graph with the transmission range $r$. As subsequent steps do not add any other edges, the almost Delaunay planar net must be a subgraph of the unit disk graph. In Step 2, each node constructs the Delaunay triangulation with their neighbors. The resulting graph after Step 2 (denoted $G_1$) includes all Delaunay edges whose lengths are at most $r$. $G_1$ can be inconsistent (that is, two nodes having differing opinions regarding their common edge) and can have edge crossing. In Step 3, we first remove all of the inconsistent edges. We then remove an illegal edge that violates the local Delaunay property for each edge crossing. After these two operations, the resulting graph becomes consistent and planar. Moreover, the removed edges in Step 3 are non-Delaunay edges, and the resulting graph contains all the edges of the Delaunay triangulation whose lengths are at most $r$. That is, the resulting graph contains the minimum spanning tree and is connected. We note that the al-
most Delaunay planar net is sometimes not even a triangulation and it may contain some non-Delaunay edges. Fig. 2 (a) shows an example of the Delaunay triangulation of 10 points and Fig. 2(b) shows the almost Delaunay planar net of the same point set. We can see that long edges are deleted in the almost Delaunay planar net and the resulting graph may have nontriangular faces. We can also see that the almost Delaunay planar net may have non-Delaunay edges (the dotted line in Fig. 2(b) is a non-Delaunay edge).

Fig. 2 (a) Delaunay triangulation of 10 points. (b) Almost Delaunay planar net of the same point set with a non-Delaunay edge (dotted line) and nontriangular faces.

At the end of this section, let us evaluate the time complexity of the algorithm to generate our underlying graph. The Voronoi polygon in Step 2 can be generated efficiently in $O(n \log n)$ time by a divide and conquer method. Because of Property 1, Step 3 can be executed in $O(m^2)$ time, where $m$ is the number of edges in the graph $G_1$. Here $m$ is $O(n)$, and so the algorithm runs in $O(n^2)$ time.

3 New Routing Strategy for Planar Graphs

In this section, we give a new routing technique. While designing the routing technique, our target is to guarantee delivery and to ensure path quality. Our routing strategy meets both criteria and can be used for any planar and connected graph. Our routing technique combines three different existing routing techniques. We first explain these three routing strategies.

**Compass Routing:** Compass routing [7] is a geographic greedy routing strategy that always relays packets to the vertex that minimizes the angle over all vertexes adjacent to the current node towards the destination node. This strategy is illustrated in Fig 3. Let $c$ be the current node with five neighbors. Packets toward the destination node $d$ will be sent to $v_2$ as it is in the closest direction to $d$. It is known that, if the underlying graph is the Delaunay triangulation, compass routing always guarantees the reachability of packets to the destination [7]. It is also known that, for some triangulations, packets may drop into never-ending loops with this strategy [7].

**Perimeter Routing:** Perimeter routing [8] is used when there are holes in the network. To bypass a hole, perimeter routing follows either the right-hand or the left-hand edges on the perimeter of the hole. Note that perimeter routing requires planarity of the underlying graph. This routing technique is often used to overcome local minima in which some greedy routing strategy is stuck. The basic strategy to use both a greedy routing and pe-
rimeter routing is explained as follows. A greedy routing strategy relays packets. If packets reach a local minimum node for the greedy strategy, it switches to perimeter routing. Perimeter routing follows the perimeter of a hole, and then the routing strategy switches again to the original greedy routing strategy when the hole is bypassed. Fig. 4 shows how the perimeter routing (with left-hand rule) bypasses a hole. In this figure, there is a hole between the current node \( c \) and the destination \( d \). We follow perimeter edges and switch to some greedy routing strategy when the current hole is bypassed.

\[ \text{Fig. 4 Bypassing a hole by perimeter routing.} \]

**Face Routing:** Face routing \([1]\) relays packets gradually to an adjacent face towards the destination node. See Fig. 5 for an explanation of face routing with the current node \( c \) and the destination \( d \). In Fig. 5, a message is routed along the interior of the faces of the underlying graph, with face changes at the edges crossing the \( c-d \) line.

\[ \text{Fig. 5 Face routing.} \]

This routing strategy has good properties for wireless sensor networks: if the underlying graph is planar, the reachability of messages from any node to the destination is guaranteed \([1]\). However, the path found by using face routing is not good as it may choose a roundabout route to the destination (see Table III in Section 4).

In our new routing strategy, we use the three routing techniques mentioned above. As mentioned in the previous paragraphs, each routing strategy has shortcomings: compass routing frequently gets stuck in local minima, perimeter routing cannot always bypass the routing hole, and face routing takes a long distance route to the destination. However, by combining these three routing strategies, we can guarantee delivery and the path length becomes satisfactory (see the proof at the end of this section and Tables I, II, and III in Section 4).

Fig. 6 shows the flowchart of our routing strategy. We start by compass routing. If the compass routing becomes stuck (no neighbor is closer to the destination than the current node), then we switch to perimeter routing. If perimeter routing makes a loop, then we switch to the face routing. In any of the states, when the distance to the destination is decreased from where we started perimeter routing, we switch back to compass routing. In the case of face routing, if we cross a critical edge then we move to a new face closer to the destination in which case we switch to perimeter routing. Here, an edge \( e \) with endpoints \( v_1 \) and \( v_2 \) is called a critical edge if it crosses the line segment \( c-d \) and none of the endpoints of \( e \) is closer to \( d \) than \( c \). This process continues until we reach to the destination.
Proof of Guaranteed Delivery: The above-mentioned routing strategy guarantees that we will finally reach the destination. Actually, face routing is sufficient to prove that we will reach the destination. In the case of face routing, we always move to a face that is closer to the destination. There are a finite number of nodes and faces in the graph, which means that we will reach the destination within a finite number of steps.

4 Experimental Evaluation of the Performance

We evaluate the proposed underlying graph (i.e., the almost Delaunay planar net) and the proposed routing strategy by computational experiments. For experiments, we generated test instances with a sensing area of 400 × 400 square units and 1000 sensor nodes located in the sensing area. Our test instances are categorized into two types: (1) sensors distributed uniformly, and (2) a hole of radius 100 units in the middle of the sensing area within which no sensors are placed. For each type of instance, we randomly generated 10 different point sets and, for each point set, we used 10 different destinations, which were decided randomly.

As explained in the introduction, selecting an appropriate pair of underlying graph and routing strategy is important. Thus, we choose some underlying graphs and routing strategies and evaluate every pair of them on our test instances. As for the underlying graph, we use three different structures, i.e., a relative neighborhood graph, a Gabriel graph, and an almost Delaunay planar net. For each graph structure, we apply compass routing [7], greedy routing [9], and our proposed routing combining compass routing, perimeter routing, and face routing.

Table I shows the success ratio for each pair of underlying graphs and routing strategies on the instances without holes (i.e., sensor nodes distributed uniformly). Here, the range means the transmission range r of sensor nodes. We note that similar types of experiments have been done in [9]. In Table I, we can see that our proposed routing strategy guarantees reachability of the destination node. We can also see that the almost Delaunay planar net with an appropriate transmission range is useful for all the three routing strategies.

<table>
<thead>
<tr>
<th>Underlying Graph</th>
<th>Compass</th>
<th>Greedy</th>
<th>Compass + Perimeter + Face</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Neighborhood Graph</td>
<td>24.23 %</td>
<td>29.70 %</td>
<td>100 %</td>
</tr>
<tr>
<td>Gabriel Graph</td>
<td>81.21 %</td>
<td>71.01 %</td>
<td>100 %</td>
</tr>
<tr>
<td>Almost Delaunay Planar Net</td>
<td>Range = 25</td>
<td>70.41 %</td>
<td>76.88 %</td>
</tr>
<tr>
<td></td>
<td>Range = 30</td>
<td>78.46 %</td>
<td>89.32 %</td>
</tr>
<tr>
<td></td>
<td>Range = 35</td>
<td>93.50 %</td>
<td>95.45 %</td>
</tr>
<tr>
<td></td>
<td>Range = 40</td>
<td>98.83 %</td>
<td>96.76 %</td>
</tr>
</tbody>
</table>

We then considered the case without assuming uniformity. We use the above-mentioned test instances with a large hole in the sensing area. Table II shows the success ratio of different routing strategies on various underlying graphs. In this table, we again can see that the messages always reach the destination with our proposed routing strategy. We can also confirm that the success ratio on this type of instance is inferior to the uniform case. From this observation, routing strategies with reachability should be useful in many realistic situations.
Finally, we discuss the quality of paths on which messages are relayed. Reachability is an important criterion to evaluate routing strategies. However, there are other important criteria to measure routing strategies, and path length is a typical one. We calculate the path lengths obtained with our routing strategy, face routing strategy [1] (which is an existing routing strategy with guaranteed reachability), and the shortest path, and then we compare them. In Table III, we can see the average path length computed by the two routing strategies on our test instances with a large hole. Here, the shortest path denotes the average of the shortest path lengths that can be computed with global information. This table shows an enhanced performance for our routing strategy using three existing routing strategies effectively.

### Table III Average path length for our routing algorithm in different underlying graphs.

<table>
<thead>
<tr>
<th>Underlying Graph</th>
<th>Face</th>
<th>Compass + Perimeter + Face</th>
<th>Shortest path</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Neighborhood Graph</td>
<td>442</td>
<td>398</td>
<td>347</td>
</tr>
<tr>
<td>Gabriel Graph</td>
<td>344</td>
<td>301</td>
<td>277</td>
</tr>
<tr>
<td>Almost Delaunay Planar Net</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range = 25</td>
<td>370</td>
<td>321</td>
<td>301</td>
</tr>
<tr>
<td>Range = 30</td>
<td>355</td>
<td>315</td>
<td>297</td>
</tr>
<tr>
<td>Range = 35</td>
<td>296</td>
<td>261</td>
<td>224</td>
</tr>
<tr>
<td>Range = 40</td>
<td>269</td>
<td>239</td>
<td>219</td>
</tr>
</tbody>
</table>

## 5 Conclusions

In this paper, we introduced an underlying graph and a routing strategy for wireless sensor networks. The underlying graph, called the almost Delaunay planar net, is a planar graph with relatively short edges. The advantages of using this graph are: (1) it is similar to the Delaunay triangulation, which has many good properties for relaying messages autonomously, and (2) it is possible to construct the graph with local computation only. The proposed routing strategy is a mixture of three existing methods (compass, perimeter, and face routings). We gave a theoretical proof of the reachability of our strategy on any connected planar graph.

We conducted computational experiments and confirmed the following properties on our proposed methods: (1) the almost Delaunay planar net is useful for both existing routing strategies and our new strategy, (2) the proposed routing strategy guarantees delivery on various planar graphs including the almost Delaunay planar net, and (3) the path length computed by our routing strategy is shorter than that with face routing, which is a known routing strategy with guaranteed reachability on planar graphs.

The effectiveness of the almost Delaunay planar net depends on choosing an appropriate value of the transmission range $r$ and this is a problem to be solved in the future. We considered only a single cluster model in this paper. Multiple clustering could further improve path quality and energy consumption.

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