Matroid matching via mixed skew–symmetric matrices

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Abstract

Tutte associates a V by V skew–symmetric matrix T, having indeterminate entries, with a graph $G = (V, E)$. This matrix, called the *Tutte matrix*, has rank exactly twice the size of a maximum cardinality matching of G. Thus, to find the size of a maximum matching it suffices to compute the rank of T. We consider the more general problem of computing the rank of $T + K$ where K is a real V by V skew–symmetric matrix. This modest generalization of the matching problem contains the linear matroid matching problem and, more generally, the linear delta–matroid parity problem. We present a tight upper bound on the rank of $T + K$ by decomposing $T + K$ into a sum of matrices whose ranks are easy to compute.

1 Introduction

Let $G = (V, E)$ be a simple graph, and let $(z_e : e \in E)$ be algebraically independent commuting indeterminates. We define a V by V skew–symmetric matrix $T = (t_{ij})$, called the *Tutte matrix* of G, such that $t_{ij} = \pm z_e$ if $ij = e \in E$, and $t_{ij} = 0$ otherwise. Tutte observed that T is nonsingular (that is, its determinant is not identically zero) if and only if G admits a perfect matching. In fact, the rank of T is equal to the size of a maximum cardinality matchable set in G. (A subset X of V is called *matchable* if $G[X]$, the subgraph induced by X, admits a perfect matching.) By applying elementary linear algebra to the Tutte matrix, Tutte proved his famous matching theorem [17]. Similar techniques prove the following extension of Tutte's theorem.

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