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Round-Robin Tournaments with a Small Number of Breaks†

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Abstract. The break minimization problem is to find a home-away assignment that minimizes the number of breaks for a given schedule of a round-robin tournament. In a recent paper, Elf et al. conjectured that this minimization problem is solvable in polynomial time if the optimal value of a given instance is less than the number of teams of the instance. We prove their conjecture affirmatively by showing that the decision problem we propose, which is related to the break minimization problem, is solvable in polynomial time. Our approach is to transform an instance of our decision problem into a collection of instances of 2SAT.

Key Words: sports scheduling, round-robin tournament, break minimization, 2SAT

1 Introduction

Sports scheduling is a significant topic of automated timetabling. Recently, a number of papers about sports scheduling have been published [1, 3–7, 9–14, 16–40], and most of them considered scheduling of a round-robin tournament.

In this paper, we consider a round-robin tournament with a home-away assignment. There are mainly two approaches to construct a schedule of such a tournament. One approach is to decide a home-away assignment of games first, then assign opponents [34, 27, 21]. The other is to fix opponents of games first and then set a home-away assignment [28, 35, 14, 26].

When we construct a schedule according to the latter method, we often encounter the problem to find a home-away assignment that minimizes the number of breaks for a fixed schedule without a home-away assignment. This problem is called the "break minimization problem." Although some enumerative algorithms, such as integer programming and constraint programming, are effective for solving this minimization problem, few theoretical results are already known. In this paper, we investigate the break minimization problem, and propose a

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polynomial time algorithm to find a home-away assignment of which the number of breaks is less than the number of teams, if it exists. Our results give the answer to the question raised in the recent paper [14] by Elf et al.

This paper consists of four sections, including this introduction. In the next section, we define the break minimization problem, and introduce a decision problem that is related to the break minimization problem. In Section 3, we propose a polynomial time algorithm to solve the decision problem. Finally, we conclude in Section 4.

2 Break Minimization

We consider a round-robin tournament of $2n$ teams with $2n-1$ slots, and denote the set of teams by $N = \{1, 2, \ldots, 2n\}$ and and that of slots by $S = \{1, 2, \ldots, 2n-1\}$. Figure 1 is an example of a schedule of six teams. In a round-robin tournament, each team plays one game in each slot, and plays every other team once.

In this paper, we assume that:
(1) each team has its home;
(2) each game is held at the home of one of the teams playing.
In a schedule, each game with '@' means that the game is held at the home of the opponent, while without '@' means that the game is held at the home of the team corresponding to the row. For example, in Fig. 1, team 4 plays team 2 at the home of team 2 in slot 3.

If a team plays two games either both at their home or both at away in slots s and $s+1$, we say that the team has a break at slot $s+1$. In this paper, a break is expressed with an underline at a slot where a break occurs. For instance, in Fig. 1, team 5 has consecutive away-games in slots 4 and 5, and we say that team 5 has a break at slot 5. In total, the schedule of Fig. 1 has six breaks. Generally, an organizer of a tournament does not prefer a schedule with many breaks. Thus, given a schedule without a home-away assignment, we are interested in finding a home-away assignment with as few breaks as possible.

Here we introduce some terms for a formal definition of the break minimization problem. We describe a possible round-robin tournament by a matrix C,

		1	2	3	4	5	(slot)
1	:	@6	@$\underline{3}$	@$\underline{5}$	2	@4	
2	:	@5	6	$\underline{4}$	@1	3	
3	:	4	$\underline{1}$	@6	5	@2	
4	:	@3	5	@2	6	$\underline{1}$	
5	:	2	@4	1	@3	@$\underline{6}$	
6	:	1	@2	3	@4	5	
(team)							

Fig. 1. Schedule of six teams, with six breaks.

	1	2	3	4	5
1 :	6	3	5	2	4
2 :	5	6	4	1	3
3 :	4	1	6	5	2
4 :	3	5	2	6	1
5 :	2	4	1	3	6
6 :	1	2	3	4	5

Fig. 2. Tournament chart of six teams.

called a *tournament chart*, whose rows and columns are indexed by N and S, respectively. The entry of C indexed by $(i, r) \in N \times S$, denoted by $c_{i,r} \in N$, represents the opponent of team i at slot r. A matrix C corresponds to a feasible round-robin tournament if and only if C satisfies the following:

(1) for each team $i \in N$, the i-th row of C is a permutation of $N \setminus \{i\}$;

(2) for any $(i, r) \in N \times S$, $c_{i',r} = i$ where $i' = c_{i,r}$.

Given a tournament chart C, an $(N \times S)$-matrix $T = (t_{i,r})$ is called a *home-away table* corresponding to C if and only if

(1) each entry of T is either H or A, and

(2) $\forall(i, r) \in N \times S$, $\{t_{i,r}, t_{i',r}\} = \{\mathrm{H}, \mathrm{A}\}$ where $i' = c_{i,r}$.

A tournament chart has many corresponding home-away tables. A pair of a tournament chart C and a corresponding home-away table T, denoted by (C, T), is called a *timetable*.

The break minimization problem is defined as follows.

Break Minimization Problem (P0)

Input: A tournament chart C of $2n$ teams.
Output: A home-away table T corresponding to C that minimizes the number of breaks of the timetable (C, T).

The tournament chart of Fig. 2 is an input of the break minimization problem. Although the timetable of Fig. 1 is a feasible solution of the instance, it is not optimal. The timetable of Fig. 3 is an optimal solution, with four breaks. Figure 4 is the home-away table corresponding to the timetable of Fig. 3.

	1	2	3	4	5
1 :	@6	3	@5	2	@4
2 :	@5	6	$\underline{4}$	@1	3
3 :	4	@1	@$\underline{6}$	5	@2
4 :	@3	5	@2	6	$\underline{1}$
5 :	2	@4	1	@3	@$\underline{6}$
6 :	1	@2	3	@4	5

Fig. 3. Optimal timetable corresponding to Fig. 2.

	1	2	3	4	5
1 :	A	H	A	H	A
2 :	A	H	<u>H</u>	A	H
3 :	H	A	<u>A</u>	H	A
4 :	A	H	A	H	<u>H</u>
5 :	H	A	H	A	<u>A</u>
6 :	H	A	H	A	H

Fig. 4. Home-Away Table corresponding to Fig. 3.

There are some previous results on the break minimization problem. Régin solved this problem with constraint programming [28], and Trick used integer programming [35]. We proposed an approximation algorithm based on positive semidefinite programming relaxation [26]. Elf et al. formulated this problem as MAX CUT on a certain kind of graphs [14], and conjectured that the break minimization problem is NP-hard. However, the complexity status of the break minimization problem is not yet determined.

The following theorem is a well-known fact in sports scheduling.

Theorem 1 (de Werra **[10]**)**.** In any timetable of $2n$ teams, the number of breaks is greater than or equal to $2n-2$.

Here we also note that every timetable has even number of breaks. When the number of breaks is less than the number of teams, the number of breaks is equal to $2n-2$.

By Theorem 1, $2n-2$ is a lower bound of the break minimization problem of $2n$ teams. Hence, if we find a home-away table that attains the lower bound, the solution is optimal. Thus, it is an interesting problem to decide whether a given tournament chart has a corresponding home-away table of which the number of breaks is equal to the lower bound. In addition, Elf et al. [14] reported that the instances of break minimization problem were solved very quickly with their algorithm when the instances had optimal solutions that attain the lower bound. They conjectured that the break minimization problem can be solved in polynomial time if a given instance of $2n$ teams has the optimal value $2n-2$.

In the next section, we prove their conjecture affirmatively by showing that the following problem can be solved in polynomial time.

<u>Decision Problem (P1)</u>

Input: A tournament chart C of $2n$ teams.
Output: A home-away table T satisfying that

(1) T is a home-away table corresponding to C, and

(2) the timetable (C,T) has $2n-2$ breaks,

if it exists; else say none exists.

3 Reduction to 2SAT

The problem k-SAT is described as follows:

> Given n Boolean variables $x_1, x_2, \ldots, x_n$, and m clauses $C_1, C_2, \ldots, C_m$ each consisting of up to k literals (either a Boolean variable x_i or its negation $\neg x_i$);
> Find a true-false assignment to the variables $x_1, x_2, \ldots x_n$ such that all clauses are satisfied if it exists, else say none exists.

It is a well-known fact that 2SAT can be solved in polynomial time [15, 2], while 3SAT (and more) is NP-complete [8]. In this section, we state that the decision problem P1 can be reduced into $2n$ instances of 2SAT and therefore solvable in polynomial time.

The following property is also well-known in sports scheduling.

Theorem 2 (de Werra **[10]).** If a timetable of $2n$ teams has $2n - 2$ breaks, then exactly two teams has no break and others have exactly one break.

From the above, it is natural to consider the following subproblems P1$_k$ ($k = 1, 2, \ldots, 2n$) related to Problem P1.

Decision Problem (P1$_k$)

Input: A tournament chart C of $2n$ teams.
Output: A home-away table T satisfying that
(1) T is a home-away table corresponding to C,
(2) the timetable (C, T) has $2n - 2$ breaks, and
(3) the k-th row of T is $(\mathrm{H}, \mathrm{A}, \mathrm{H}, \mathrm{A}, \ldots, \mathrm{H})$,
if it exists; else say none exists.

By Theorem 2, Problem P1 is feasible if and only if at least one of those subproblems P1$_1$, P1$_2$, ..., and P1$_{2n}$ is feasible.

In the following, we formulate Problem P1$_k$ as an instance of 2SAT. We represent a given tournament chart C by using the notation s_{ij} $(i \neq j)$ that shows the slot when the game between teams i and j is held. Note that $s_{ij} = s_{ji}$. By Condition (3) of Problem P1$_k$, the home-away assignment of all the slots of team k is fixed. Accordingly, for each team $i \in N \setminus \{k\}$, the home-away assignment of slot s_{ki} is also determined. In the following, we introduce a *temporary home-away table* $T^k = (t^k_{i,r})$ whose rows and columns are indexed by N and S, respectively. The temporary home-away table T^k satisfies that:

(1) k-th row of T^k is $(\mathrm{H}, \mathrm{A}, \mathrm{H}, \mathrm{A}, \ldots, \mathrm{H})$;

(2) $\forall i \in N \setminus \{k\}$, if s_{ki} is even, then i-th row of T^k satisfies that

$$(t^k_{i,s_{ki}}, t^k_{i,s_{ki}+1}, \ldots, t^k_{i,2n-1}, t^k_{i,1}, t^k_{i,2}, \ldots, t^k_{i,s_{ki}-1}) = (\mathrm{H}, \mathrm{A}, \mathrm{H}, \mathrm{A}, \ldots, \mathrm{H});$$

(3) $\forall i \in N \setminus \{k\}$, if s_{ki} is odd, then i-th row of T^k satisfies that

$$(t^k_{i,s_{ki}}, t^k_{i,s_{ki}+1}, \ldots, t^k_{i,2n-1}, t^k_{i,1}, t^k_{i,2}, \ldots, t^k_{i,s_{ki}-1}) = (\mathrm{A}, \mathrm{H}, \mathrm{A}, \mathrm{H}, \ldots, \mathrm{A}).$$

It should be noted that the temporary assignment is uniquely determined, and is inconsistent to a given tournament chart unless $2n = 2$.

We introduce Boolean variables $x_{i,r}$ ($(i,r) \in N \setminus \{k\} \times S$). Assume that Problem P1$_k$ has a solution (a home-away table) T^*. We represent the home-away table T^* by the following true-false assignment $x^*_{i,r} \in \{\texttt{TRUE}, \texttt{FALSE}\}$ ($(i,r) \in N \setminus \{k\} \times S$) defined by

$$x^*_{i,r} = \begin{cases} \texttt{TRUE} & (t^*_{i,r} = t^k_{i,r}), \\ \texttt{FALSE} & (t^*_{i,r} \neq t^k_{i,r}). \end{cases}$$

Then, the true-false assignment satisfies the following three properties.

(a) For each team $i \in N \setminus \{k\}$, $x^*_{i,s_{ki}} = \texttt{TRUE}$.

(b) Since T^* is consistent to the given tournament chart, for any pair of distinct teams $i, j \in N \setminus \{k\}$,

$$[t^k_{i,s_{ij}} \neq t^k_{j,s_{ij}} \implies x^*_{i,s_{ij}} = x^*_{j,s_{ij}}] \quad \text{and} \quad [t^k_{i,s_{ij}} = t^k_{j,s_{ij}} \implies x^*_{i,s_{ij}} \neq x^*_{j,s_{ij}}].$$

(c) Since each team $i \in N \setminus \{k\}$ has at most one break, the values of the variables

$$(x^*_{i,s_{ki}}, x^*_{i,s_{ki}+1}, \ldots, x^*_{i,2n-1}, x^*_{i,1}, \ldots, x^*_{i,s_{ki}-1})$$

is equal to one of the following:
(TRUE, TRUE, ..., TRUE, TRUE), (TRUE, TRUE, ..., TRUE, FALSE),
(TRUE, TRUE, ..., FALSE, FALSE), ..., (TRUE, FALSE, ..., FALSE, FALSE).

It is easy to see that Property (c) is equivalent to the condition that for any team $i \in N \setminus \{k\}$, all the clauses $(x^*_{i,s_{ki}} \vee \neg x^*_{i,s_{ki}+1})$, $(x^*_{i,s_{ki}+1} \vee \neg x^*_{i,s_{ki}+2}), \ldots,$ $(x^*_{i,2n-1} \vee \neg x^*_{i,1}), (x^*_{i,1} \vee \neg x^*_{i,2}), \ldots, (x^*_{i,s_{ki}-2} \vee \neg x^*_{i,s_{ki}-1})$ are satisfied.

Now we formulate Problem P1$_k$ as 2SAT.

Satisfiability Problem (2SAT$_k$)

Input: A tournament chart C of $2n$ teams.

Output: A true-false assignment $x_{i,r} \in \{\texttt{TRUE}, \texttt{FALSE}\}$ ($(i,r) \in N \setminus \{k\} \times S$) satisfying the conditions that

(a) $x_{i,s_{ki}} = \texttt{TRUE}$ $\quad (i \in N \setminus \{k\})$,

(b1) $x_{i,s_{ij}} = x_{j,s_{ij}}$ $\quad (\{i,j\} \subseteq N \setminus \{k\}, t^k_{i,s_{ij}} \neq t^k_{j,s_{ij}})$,

(b2) $x_{i,s_{ij}} \neq x_{j,s_{ij}}$ $\quad (\{i,j\} \subseteq N \setminus \{k\}, t^k_{i,s_{ij}} = t^k_{j,s_{ij}})$,

(c) $(x_{i,s_{ki}} \vee \neg x_{i,s_{ki}+1}) = (x_{i,s_{ki}+1} \vee \neg x_{i,s_{ki}+2})$
$= \cdots = (x_{i,2n-1} \vee \neg x_{i,1}) = (x_{i,1} \vee \neg x_{i,2})$
$= \cdots = (x_{i,s_{ki}-2} \vee \neg x_{i,s_{ki}-1}) = \texttt{TRUE}$ $\quad (i \in N \setminus \{k\})$

if it exists; else say none exists.

If we have a true-false assignment $x'_{i,r} \in \{\texttt{TRUE}, \texttt{FALSE}\}$ ($(i,r) \in N \setminus \{k\} \times S$) satisfying all the above conditions, we can construct a home-away table $T' = (t'_{i,r})$ satisfying that the k-th row of T' is (H, A, H, A, ..., H) and for any $i \in N \setminus \{k\}$,

$$t'_{i,r} = \begin{cases} t^k_{i,r} & (x'_{i,r} = \texttt{TRUE}), \\ \text{A} & (x'_{i,r} = \texttt{FALSE} \text{ and } t^k_{i,r} = \text{H}), \\ \text{H} & (x'_{i,r} = \texttt{FALSE} \text{ and } t^k_{i,r} = \text{A}). \end{cases}$$

From Conditions (a,b1,b2), it is clear that the home-away table T' is consistent to the given tournament chart C. Condition (c) implies that for each team, the number of breaks is at most one. Since team k has no break, the total number of breaks is equal to $2n-2$.

Conversely, we already saw that if we have a home-away table T^* satisfying all the conditions of Problem P1$_k$, we can construct a true-false assignment $x^*_{i,r}$ satisfying Conditions (a,b1,b2,c).

Each of the problems 2SAT$_1$, 2SAT$_2$, …, and 2SAT$_{2n}$ is a special case of 2SAT and therefore solvable in polynomial time. Consequently, Problem P1 is solvable in polynomial time.

4 Conclusion

In this paper, we considered the break minimization problem of a round-robin tournament, and proposed a related decision problem. We showed that, by reduction to 2SAT, there is a polynomial time algorithm for finding a home-away assignment of which the number of breaks is less than the number of teams, if it exists. Our results gave the answer to the question raised in a recent paper [14] by Elf et al.

We also have the following problem.

Decision Problem (P2)

Input: A tournament chart C of $2n$ teams.
Output: A home-away table T satisfying that
(1) T is a home-away table corresponding to C,
(2) the timetable (C, T) satisfies that each team has exactly one break.
if it exists; else say none exists.

A timetable satisfying Condition (2) of Problem P2 is called "equitable," and sometimes it is preferred to a timetable with $2n-2$ breaks. With a few modifications to the proposed procedure, we can show that Problem P2 is also solvable in polynomial time.

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